

Semi-Leptonic B Meson Decays to Excited D Mesons in the Covariant Oscillator Quark Model

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The spectra and branching ratios of the weak semi-leptonic B meson decays to the first excited D mesons are predicted, taking into account the confined effects of quarks using the covariant oscillator quark model (COQM). In the COQM the same relation between general weak transition form factors as that in HQET is derived, and the concrete form of the Isgur-Wise function is given with no free parameters. Our results are somewhat different from those of other models. The present experimental data are not sufficient for comparison.

§1. Introduction

The decay of B mesons has been one of the most important topics of high energy physics for many years, and its theoretical explanation is a matter of great urgency, since now a large number of B mesons are produced in B -factory experiments. However, it has been a difficult task to predict the spectra and widths of such decays quantitatively, since they are largely affected by the confined effects of quarks. Among many possible channels, an analysis of the $B \rightarrow D^* l \bar{\nu}_l$ and $B \rightarrow D l \bar{\nu}_l$ decays is one of the main interests in the heavy quark effective theory (HQET).¹⁾ In HQET, all generally independent form factors, appearing in the effective meson transition current $J_\mu^{B \rightarrow D^*/D}$, are represented by one universal form factor (FF) function (the Isgur-Wise function $\xi(\omega)$), thus leading to various FF-relations among them. The value of $\xi(\omega)$ at the zero-recoil point is to be unity [$\xi(1) = 1$] in the heavy quark mass limit $m_Q \rightarrow \infty$, reflecting the conserved charge of heavy quark symmetry (HQS). This function $\xi(\omega)$ describes the confined effects of quarks. However, HQET and/or HQS themselves are not able to predict the concrete form of $\xi(\omega)$, and thus they cannot describe the FF function and the decay spectra in all regions of q^2 . For this, it is, in principle, necessary to know covariant wave functions (WF) of mesons for both spin and space-time variables, and presently we are required to resort to some models with a covariant framework.

We use the covariant oscillator quark model (COQM)²⁾ in order to estimate the confined effects of quarks. One of the most important motives for this model is to describe covariantly the center-of-mass motion of hadrons, while preserving the considerable success of the non-relativistic quark model regarding the static properties

of hadrons. A keystone in COQM for doing this is directly treating the squared masses of hadrons, rather than the mass itself, as done in conventional approaches. This makes the covariant treatment simple. The COQM has a long history of development,³⁾ and its origin is traced back to the bilocal theory of Yukawa.⁴⁾ The COQM has been applied to various problems⁵⁾ with satisfactory results. Both of the covariant space-time and spin WF of general meson systems are determined from the analyses of static problems such as mass spectra⁶⁾ and radiative transitions.⁷⁾ Recently, Ishida et al.^{8), 9)} have studied the weak decays of heavy hadrons using this model and derived the same relations of weak form factors for the heavy-to-heavy transition as those derived with HQET.¹⁾ In addition, our model is also applicable to heavy-to-light transitions. As a consequence, this model does incorporate the features of heavy quark symmetry and can be used to compute the form factors for heavy-to-light transitions as well, which is beyond the scope of HQET. Actually, in previous papers we made analyses of the spectra of exclusive semi-leptonic^{9), 10)} decays of B -mesons, of non-leptonic decays of B mesons,¹¹⁾ of hadronic weak decays of Λ_b baryons,¹²⁾ and of rare radiative decays of B mesons¹³⁾ and Λ_b baryons,¹⁴⁾ leading to encouraging results. In this paper we extend the application of our model to weak semi-leptonic decays of B mesons to the first excited D mesons, D^{**} . It should be noted that the description by HQET of this decay process into excited mesons is less reliable than that of decays to ground D/D^* mesons, since the approximation in the HQS limit, where the momentum of the light degrees of freedom is ignored in comparison with m_Q , is worse in the excited D mesons than in the ground D mesons. In contrast with this situation, the description with COQM is expected to be reliable, as discussed in the third work of Ref. 2), also in this case.

§2. Model Framework, form factors and decay width

of $B \rightarrow (D_2^*, D_1^{j_q=3/2}, D_{1*}^{j_q=1/2}, D_0^*) l \bar{\nu}_l$

2.1. Wave functions in COQM

The general treatment of COQM may be called the “boosted LS -coupling scheme,”²⁾ where the wave functions (WF) are tensors in $\tilde{U}(4) \times O(3,1)$ -space and reduce to those in $SU(2)_{\text{spin}} \times O(3)_{\text{orbit}}$ -space in the nonrelativistic quark model in the hadron rest frame. The spinor and space-time portion of the WF separately satisfy the respective covariant equations, the Bargmann-Wigner (BW) equation for the former and the covariant oscillator equation for the latter. The form of the meson WF has been determined completely through the analysis of the mass spectra.

In COQM, the meson states are described by bi-local fields $\Phi_A^B(x_{1\mu}, x_{2\mu})$, where $x_{1\mu}(x_{2\mu})$ is the space-time coordinate of the constituent quark (antiquark), and $A = (a, \alpha)$ ($B = (b, \beta)$) describes its flavor and covariant spinor. Here we write only the positive frequency part of the relevant ground states:

$$\Phi_{P,A}^B(x_{1\mu}, x_{2\mu}) = e^{iP \cdot X} U(P)_A^B f_\beta(x_\mu; P), \quad (2.1)$$

where U and f_β are the covariant spinor and internal space-time WF, respectively, satisfying the Bargmann-Wigner and oscillator wave equations. The quantity $x_\mu(X_\mu)$

is the relative (CM) coordinate, $x_\mu \equiv x_{1\mu} - x_{2\mu}$ ($X_\mu \equiv (m_1 x_{1\mu} + m_2 x_{2\mu})/(m_1 + m_2)$), where the m_i represent the quark masses). The function U is given by

$$U(P) = \frac{1}{2\sqrt{2}} [(-\gamma_5 P_s(v) + i\gamma_\mu V_\mu(v))(1 + iv \cdot \gamma)], \quad (2.2)$$

where $P_s(V_s)$ represents the pseudoscalar (vector) meson field, and $v_\mu \equiv P_\mu/M$ [$P_\mu(M)$ is the four momentum (mass) of the meson]. The function U , being represented by the direct product of quark and antiquark Dirac spinors with the meson velocity, is reduced to the non-relativistic Pauli-spin function in the meson rest frame. The function f_β is given^{*)} by

$$f_\beta(x_\mu; P) = \frac{\beta}{\pi} \exp\left(-\frac{\beta}{2} (x_\mu^2 + 2(v \cdot x)^2)\right); \quad \beta = \sqrt{\mu K}, \quad (2.3)$$

where μ is the reduced mass, and K is the spring constant of oscillator potential. The quantity β represents the inverse “size” of WF.

The WF of D^{**} mesons are obtained by operating with the creation operator $a_\lambda^\dagger = (1/\sqrt{2\beta'}) (\beta' x_\lambda - \partial/\partial x_\lambda)$ on the ground state WF as

$$\begin{aligned} \Phi_{P'}(x_{1\mu}, x_{2\mu}) &= \frac{1}{2\sqrt{2}} (\epsilon_{\nu\lambda}^{(h)}(v') i\gamma_\nu - \epsilon_\lambda^{(h)}(v') \gamma_5) (1 + iv' \cdot \gamma) e^{iP' \cdot X'} a_\lambda^\dagger f_{\beta'}(x_\mu; P') \\ &= \frac{1}{2\sqrt{2}} (\epsilon_{\nu\lambda}^{(h)}(v') i\gamma_\nu - \epsilon_\lambda^{(h)}(v') \gamma_5) (1 + iv' \cdot \gamma) \\ &\quad \times e^{iP' \cdot X'} \sqrt{2\beta'} (x_\lambda + v'_\lambda v' \cdot x) f_{\beta'}(x_\mu; P'), \end{aligned} \quad (2.4)$$

where $\epsilon_{\nu\lambda}^{(h)}(v')$ ($\epsilon_\lambda^{(h)}(v')$) represents the polarization tensor of the final D^{**} meson in $^3P_{J=2,1,0}$ ($^1P_{J=1}$) states, and β' is the inverse size of the final D meson system. The Pauli-conjugate of WF is defined by $\bar{\Phi}_{P'} \equiv -\gamma_4 \Phi_{P'}^\dagger \gamma_4$. When the final D mesons are to move in the z -direction with velocity $v'_\mu = (0, 0, \omega_3, i\omega)$ ($\omega_3 \equiv \sqrt{\omega^2 - 1}$), the conjugate of the polarization tensors are given by

$$\begin{aligned} \text{For } ^1P_1, \quad \tilde{\epsilon}^{(\pm)}(v') &= \frac{\mp 1}{\sqrt{2}} (1, \mp i, 0, i0), \quad \tilde{\epsilon}^{(0)}(v') = (0, 0, \omega, i\omega_3) \\ \text{For } ^3P_2, \quad \tilde{\epsilon}_{\nu\lambda}^{(\pm 2)}(v') &= \tilde{\epsilon}_\nu^{(\pm)} \tilde{\epsilon}_\lambda^{(\pm)}, \quad \tilde{\epsilon}_{\nu\lambda}^{(\pm 1)}(v') = \frac{1}{\sqrt{2}} (\tilde{\epsilon}_\nu^{(\pm)} \tilde{\epsilon}_\lambda^{(0)} + \tilde{\epsilon}_\nu^{(0)} \tilde{\epsilon}_\lambda^{(\pm)}) \\ \tilde{\epsilon}_{\nu\lambda}^{(0)}(v') &= \frac{1}{\sqrt{6}} (\tilde{\epsilon}_\nu^{(+)} \tilde{\epsilon}_\lambda^{(-)} + 2\tilde{\epsilon}_\nu^{(0)} \tilde{\epsilon}_\lambda^{(0)} + \tilde{\epsilon}_\nu^{(-)} \tilde{\epsilon}_\lambda^{(+)}) \\ \text{For } ^3P_1, \quad \tilde{\epsilon}_{\nu\lambda}^{(\pm 1)}(v') &= \frac{\pm 1}{\sqrt{2}} (\tilde{\epsilon}_\nu^{(\pm)} \tilde{\epsilon}_\lambda^{(0)} - \tilde{\epsilon}_\nu^{(0)} \tilde{\epsilon}_\lambda^{(\pm)}) = \frac{1}{\sqrt{2}} \epsilon_{\nu\lambda\alpha\beta} v'_\alpha \tilde{\epsilon}_\beta^{(\pm)} \\ \tilde{\epsilon}_{\nu\lambda}^{(0)}(v') &= \frac{1}{\sqrt{2}} (\tilde{\epsilon}_\nu^{(+)} \tilde{\epsilon}_\lambda^{(-)} - \tilde{\epsilon}_\nu^{(-)} \tilde{\epsilon}_\lambda^{(+)}) = \frac{1}{\sqrt{2}} \epsilon_{\nu\lambda\alpha\beta} v'_\alpha \tilde{\epsilon}_\beta^{(0)} \\ \text{For } ^3P_0, \quad \tilde{\epsilon}_{\nu\lambda}(v') &= \frac{1}{\sqrt{3}} (\tilde{\epsilon}_\nu^{(+)} \tilde{\epsilon}_\lambda^{(-)} - \tilde{\epsilon}_\nu^{(0)} \tilde{\epsilon}_\lambda^{(0)} + \tilde{\epsilon}_\nu^{(-)} \tilde{\epsilon}_\lambda^{(+)}) = -\frac{\delta_{\nu\lambda} + v'_\nu v'_\lambda}{\sqrt{3}}. \end{aligned} \quad (2.5)$$

^{*)} In this paper we employ the pure-confining approximation, neglecting the effect of the one-gluon-exchange potential, which is expected to be good for the heavy/light-quark meson system, since this system has comparatively large space extension (see ref. 6)).

2.2. Effective weak current

The effective weak interaction describing the $B \rightarrow D^{**} + W$ process is given by the covariant overlapping of the initial and final meson WF,

$$\mathcal{L}_W = \frac{g}{2\sqrt{2}} V_{cb} \int d^4 x_1 \int d^4 x_2 \langle \sqrt{2M'} \bar{\Phi}_{P'}(X', x) i\gamma_\mu (1 + \gamma_5) \sqrt{2M} \Phi_P(X, x) \rangle W_\mu(x_1),$$

where we denote the momenta (masses) of the initial B and final D^{**} mesons, respectively, as $P_\mu(M)$ and $P'_\mu(M')$. The momentum of the emitted W -boson is denoted as q_μ . The effective weak transition current $J_\mu (= V_\mu + A_\mu)$ is defined by the equation

$$\begin{aligned} \mathcal{L}_W &= \frac{g}{2\sqrt{2}} V_{cb} \int d^4 X (1/i) J_\mu(X) W_\mu(X) \\ &\propto V_{cb} (2\pi)^4 \delta^{(4)}(P - P' - q) (1/i) J_\mu(P', P) W_\mu(q), \end{aligned} \quad (2.6)$$

where $J_\mu(P', P) \equiv J_\mu(X = 0)$. The overlapping of WF is taken separately in the spinor part and the space-time part, and the final form of $J_\mu(X)$ is given by

$$\begin{aligned} (1/i) J_\mu(X) &= \sqrt{2M} \sqrt{2M'} O_{\mu\lambda}^s O_\lambda^x \\ O_{\mu\lambda}^s &= \left\langle \frac{-1}{2\sqrt{2}} (1 + iv' \cdot \gamma) (i\gamma_\nu \tilde{\epsilon}_{\nu\lambda}^{(h)} + \gamma_5 \tilde{\epsilon}_\lambda^{(h)}) i\gamma_\mu (1 + \gamma_5) \frac{-1}{2\sqrt{2}} \gamma_5 (1 + iv \cdot \gamma) \right\rangle \\ &= \frac{1}{2} \{ \tilde{\epsilon}_{\nu\lambda}^{(h)} (\varepsilon_{\nu\mu\alpha\beta} v_\alpha v'_\beta - (\omega + 1) \delta_{\nu\mu} + v'_\nu v_\mu - v'_\mu v_\nu) + \tilde{\epsilon}_\lambda^{(h)} (v_\mu + v'_\mu) \} \\ O_\lambda^x &= \int d^4 x e^{-iP' \cdot X'} f_{\beta'}(x, P') \sqrt{2\beta'} (x_\lambda + v'_\lambda (v' \cdot x)) f_\beta(x, P) e^{iP \cdot X} e^{-iq \cdot x_1} \\ &= -iF \times (v_\lambda - \omega v'_\lambda) e^{i(P - P' - q) \cdot X}, \end{aligned} \quad (2.7)$$

where

$$\begin{aligned} F &= \sqrt{2\beta'} \frac{m_d}{C} \left\{ -(\beta - \beta') \frac{M}{M_0} + 2\beta\omega \frac{M'}{M'_0} \right\} I_{HO}(\omega), \\ C &= (\beta - \beta')^2 + 4\beta\beta'\omega^2 \\ I_{HO}(\omega) &= \frac{4\beta\beta'}{\beta + \beta'} \frac{1}{\sqrt{C}} \exp \frac{-m_d^2}{2C} \left[(\beta + \beta') \left\{ \left(\frac{M}{M_0} \right)^2 + \left(\frac{M'}{M'_0} \right)^2 - 2\omega \frac{MM'}{M_0 M'_0} \right\} \right. \\ &\quad \left. + 2(\omega^2 - 1) \left\{ \beta \left(\frac{M'}{M'_0} \right)^2 + \beta' \left(\frac{M}{M_0} \right)^2 \right\} \right]. \\ (M_0 &= m_b + m_d, \quad M'_0 = m_c + m_d) \end{aligned} \quad (2.8)$$

The function F here describes the confined effects of quarks in the relevant processes.

2.3. Form factor relation

The HQET suggests that the angular momentum of the light degrees of freedom, \mathbf{j}_q , obtained by the composition of the orbital angular momentum \mathbf{L} and the spin of the light quark \mathbf{S}_q as $\mathbf{j}_q = \mathbf{L} + \mathbf{S}_q$, is approximately a good quantum number, and the states of D^{**} with definite $j_q = 3/2$ and $1/2$ quantum numbers, $D^{j_q=3/2}$ and $D^{j_q=1/2}$, are expected to be realized in nature. The values of the total J of $j_q = 3/2$

states are $J = 2, 1$ and those of $j_q = 1/2$ are $J = 1, 0$. The $J = 2, 0$ states correspond to the ${}^3P_{2,0}$ states in the boosted LS -coupling scheme. On the other hand, the $J = 1$ states with $j_q = 3/2$ and $1/2$ are represented by the superposition of 3P_1 and 1P_1 states as¹⁾

$$\begin{aligned} D_1^{j_q=3/2} &= -\sqrt{\frac{1}{3}}D^{{}^3P_1} - \sqrt{\frac{2}{3}}D^{{}^1P_1}, \\ D_{1*}^{j_q=1/2} &= \sqrt{\frac{2}{3}}D^{{}^3P_1} - \sqrt{\frac{1}{3}}D^{{}^1P_1}. \end{aligned} \quad (2.9)$$

By taking this superposition into account, the effective currents J_μ , Eq. (2.7), for the respective D^{**} mesons can be rewritten as

$$\begin{aligned} J_\mu^{B \rightarrow D_2^*} / \sqrt{MM'} &= F(\omega) \tilde{\epsilon}_{\nu\lambda}^{(h)} v_\lambda (\epsilon_{\nu\mu\alpha\beta} v_\alpha v'_\beta - (\omega + 1) \delta_{\nu\mu} - v'_\mu v_\nu) \\ J_\mu^{B \rightarrow D_1^{j_q=3/2}} / \sqrt{MM'} &= F(\omega) \frac{-1}{\sqrt{6}} \left[\tilde{\epsilon} \cdot v (3v_\mu + (2 - \omega)v'_\mu) \right. \\ &\quad \left. + (1 - \omega^2) \tilde{\epsilon}_\mu - (\omega + 1) \epsilon_{\mu\lambda\alpha\beta} v_\lambda v'_\alpha \tilde{\epsilon}_\beta \right] \\ J_\mu^{B \rightarrow D_{1*}^{j_q=1/2}} / \sqrt{MM'} &= F(\omega) \frac{1}{\sqrt{3}} \left[-\tilde{\epsilon} \cdot v (\omega + 1) v'_\mu \right. \\ &\quad \left. + (1 - \omega^2) \tilde{\epsilon}_\mu - (\omega + 1) \epsilon_{\mu\lambda\alpha\beta} v_\lambda v'_\alpha \tilde{\epsilon}_\beta \right] \\ J_\mu^{B \rightarrow D_0^*} / \sqrt{MM'} &= F(\omega) \frac{1}{\sqrt{3}} (\omega + 1) (v_\mu - v'_\mu). \end{aligned} \quad (2.10)$$

The general weak transition form factors are defined by

$$\begin{aligned} J_\mu^{B \rightarrow D_2^*} / \sqrt{MM'} &= \tilde{h} \epsilon_{\mu\alpha\beta\gamma} \tilde{\epsilon}_{\alpha\nu} v_\nu (v + v')_\beta (v - v')_\gamma - \tilde{k} \tilde{\epsilon}_{\mu\nu} v_\nu \\ &\quad + \tilde{\epsilon}_{\alpha\beta} v_\alpha v_\beta \left[\tilde{b}_+(v + v')_\mu + \tilde{b}_-(v - v')_\mu \right] \\ J_\mu^{B \rightarrow D_1^{j_q=3/2}} / \sqrt{MM'} &= \tilde{l}_{3/2} \tilde{\epsilon}_\mu - \tilde{\epsilon} \cdot v \left[\tilde{c}_{3/2+} (v + v')_\mu + \tilde{c}_{3/2-} (v - v')_\mu \right] \\ &\quad - \tilde{q}_{3/2} \epsilon_{\mu\alpha\beta\gamma} \tilde{\epsilon}_\alpha (v + v')_\beta (v - v')_\gamma \\ J_\mu^{B \rightarrow D_{1*}^{j_q=1/2}} / \sqrt{MM'} &= \tilde{l}_{1/2} \tilde{\epsilon}_\mu - \tilde{\epsilon} \cdot v \left[\tilde{c}_{1/2+} (v + v')_\mu + \tilde{c}_{1/2-} (v - v')_\mu \right] \\ &\quad - \tilde{q}_{1/2} \epsilon_{\mu\alpha\beta\gamma} \tilde{\epsilon}_\alpha (v + v')_\beta (v - v')_\gamma \\ J_\mu^{B \rightarrow D_0^*} / \sqrt{MM'} &= \tilde{u}_+ (v + v')_\mu + \tilde{u}_- (v - v')_\mu. \end{aligned} \quad (2.11)$$

By comparing Eq. (2.10) with Eq. (2.11), we can derive the following relation between the form factors:

$$\begin{aligned} \tilde{h} &= -\tilde{b}_+ = \tilde{b}_- = (1/2)F(\omega), & \tilde{k} &= (\omega + 1)F(\omega), \\ \tilde{l}_{3/2} &= ((\omega^2 - 1)/\sqrt{6})F(\omega), \\ \tilde{c}_{3/2+} &= ((5 - \omega)/(2\sqrt{6}))F(\omega), & \tilde{c}_{3/2-} &= \tilde{q}_{3/2} = ((1 + \omega)/(2\sqrt{6}))F(\omega), \\ \tilde{l}_{1/2} &= ((1 - \omega^2)/\sqrt{3})F(\omega), & \tilde{c}_{1/2+} &= -\tilde{c}_{1/2-} = -\tilde{q}_{1/2} = ((\omega + 1)/(2\sqrt{3}))F(\omega), \\ \tilde{u}_+ &= 0, & \tilde{u}_- &= ((\omega + 1)/\sqrt{3})F(\omega). \end{aligned} \quad (2.12)$$

The above relations are the same as those obtained with HQET.^{15),16)} In HQET, the $B \rightarrow D^{**}$ current is described by two independent Isgur-Wise functions, $\tau_{3/2}(\omega)$ and $\tau_{1/2}(\omega)$, whose concrete forms are not derivable from HQET. In our scheme they are represented by a single overlapping F function Eq. (2.8) as

$$\tau_{3/2}(\omega) = F(\omega), \quad \tau_{1/2}(\omega) = F(\omega) (\omega + 1)/\sqrt{3}. \quad (2.13)$$

Calculations to derive these equations are given in Appendix A.

2.4. Helicity amplitude and decay spectra

Helicity amplitudes H_h are defined by decomposing $J_\mu(X=0)$ into W -boson polarization vectors $e_\mu^{(h)}(q)$ as

$$(1/i)J_\mu(X=0) = i(H_+e_\mu^{(+)}(q) + H_-e_\mu^{(-)}(q) + H_0e_\mu^{(0)}(q) - H_se_\mu^{(s)}(q)). \quad (2.14)$$

Thus, H_h is obtained as $H_h = \tilde{e}_\mu^{(h)}(q) J_\mu$. By supposing that the W -boson moves in the $-z$ direction with momentum $q_\mu = (0, 0, -|\mathbf{q}|, iq_0) = (0, 0, -M'\omega_3, i(M - M'\omega))$ in the rest frame of the initial B meson, the conjugate of the polarization vectors are given by

$$\tilde{e}_\mu^{(\pm)}(q) = \frac{\pm 1}{\sqrt{2}}(1, \pm i, 0, i0), \quad \tilde{e}_\mu^{(0)}(q) = \frac{1}{\sqrt{-q^2}}(0, 0, -q_0, i|\mathbf{q}|), \quad \tilde{e}_\mu^{(s)}(q) = \frac{1}{\sqrt{-q^2}}q_\mu.$$

The actual forms of the helicity amplitudes describing the decay into the D^{**} mesons in the $^{2S+1}L_J$ state, $H_h^{2S+1L_J}$, are given by

$$\begin{aligned} H_\pm^{3P_2} &= \sqrt{MM'}F \frac{-\omega_3}{\sqrt{2}}(\mp\omega_3 + \omega + 1), \\ H_0^{3P_2} &= -\sqrt{\frac{2}{3}}\sqrt{MM'}F(M - M')\frac{\omega_3(\omega + 1)}{\sqrt{-q^2}}, \\ H_s^{3P_2} &= -\sqrt{\frac{2}{3}}\sqrt{MM'}F(M + M')\frac{\omega_3^2}{\sqrt{-q^2}}, \\ H_\pm^{3P_1} &= \pm\sqrt{MM'}F \frac{-\omega_3}{\sqrt{2}}(\mp\omega_3 + \omega + 1), \quad H_{0,s}^{3P_1} = 0 \\ H_\pm^{1P_1} &= 0, \quad H_0^{1P_1} = \sqrt{MM'}F(M + M')\frac{\omega_3^2}{\sqrt{-q^2}}, \\ H_s^{1P_1} &= \sqrt{MM'}F(M - M')\frac{\omega_3(\omega + 1)}{\sqrt{-q^2}}, \\ H_\pm^{3P_0} &= 0, \quad H_0^{3P_0} = \frac{1}{\sqrt{3}}\sqrt{MM'}F(M - M')\frac{\omega_3(\omega + 1)}{\sqrt{-q^2}}, \\ H_s^{3P_0} &= \frac{1}{\sqrt{3}}\sqrt{MM'}F(M + M')\frac{\omega_3^2}{\sqrt{-q^2}}. \end{aligned} \quad (2.15)$$

By taking into account the superposition of Eq. (2.9), the helicity amplitudes describing the decays into the states with definite j_q quantum numbers are given by

$$H_\pm^{D_1^{j_q=3/2}} = -\sqrt{\frac{1}{3}}H_\pm^{3P_1}, \quad H_{0,s}^{D_1^{j_q=3/2}} = -\sqrt{\frac{2}{3}}H_{0,s}^{1P_1},$$

$$H_{\pm}^{D_{1*}^{jq=1/2}} = \sqrt{\frac{2}{3}} H_{\pm}^3 P_1, \quad H_{0,s}^{D_{1*}^{jq=1/2}} = -\sqrt{\frac{1}{3}} H_{0,s}^1 P_1. \quad (2.16)$$

Decay spectra are obtained by using these H_h through the usual procedure:

$$\begin{aligned} \frac{d\Gamma}{dq^2} &= \frac{|V_{cb}|^2 G_F^2}{192\pi^3 M^2} |\mathbf{p}'|^2 \left(1 + \frac{m_l^2}{q^2}\right)^2 (-q^2) \\ &\times \left[\left(2 - \frac{m_l^2}{q^2}\right) (|H_+|^2 + |H_-|^2 + |H_0|^2) - 3 \frac{m_l^2}{q^2} |H_s|^2 \right], \end{aligned} \quad (2.17)$$

where m_l is the mass of the lepton and G_F ($= 1.166392 \times 10^{-5} \text{ GeV}^{-2}$) is the Fermi coupling constant.

The results for the respective D^{**} mesons are given by

$$\begin{aligned} \frac{d\Gamma^{D_2^*}}{dq^2} &= \frac{|V_{cb}|^2 G_F^2 M M'^2}{96\pi^3} \omega_3 \left(1 + \frac{m_l^2}{q^2}\right)^2 F(\omega)^2 \left[\left(1 - \frac{m_l^2}{2q^2}\right) \omega_3^2 (\omega + 1)^2 \right. \\ &\times \left. \left\{ 2\omega \frac{1 + r^2 - 2r\omega}{\omega + 1} + \frac{2}{3}(1 - r)^2 \right\} + m_l^2 \frac{\omega_3^4 (1 + r)^2}{-q^2} \right]. \end{aligned} \quad (2.18)$$

$$\begin{aligned} \frac{d\Gamma^{D_1^{jq=3/2}}}{dq^2} &= \frac{|V_{cb}|^2 G_F^2 M M'^2}{96\pi^3} \omega_3 \left(1 + \frac{m_l^2}{q^2}\right)^2 F(\omega)^2 (\omega + 1)^2 \left[\left(1 - \frac{m_l^2}{2q^2}\right) \frac{2}{3} (\omega - 1)^2 \right. \\ &\times \left. \left\{ \omega \frac{1 + r^2 - 2r\omega}{\omega - 1} + (1 + r)^2 \right\} + m_l^2 \frac{\omega_3^2 (1 - r)^2}{-q^2} \right]. \end{aligned} \quad (2.19)$$

$$\begin{aligned} \frac{d\Gamma^{D_1^{jq=1/2}}}{dq^2} &= \frac{|V_{cb}|^2 G_F^2 M M'^2}{96\pi^3} \omega_3 \left(1 + \frac{m_l^2}{q^2}\right)^2 F(\omega)^2 \frac{(\omega + 1)^2}{3} \left[\left(1 - \frac{m_l^2}{2q^2}\right) (\omega - 1)^2 \right. \\ &\times \left. \left\{ 4\omega \frac{1 + r^2 - 2r\omega}{\omega - 1} + (1 + r)^2 \right\} + m_l^2 \frac{3\omega_3^2 (1 - r)^2}{-2q^2} \right]. \end{aligned} \quad (2.20)$$

$$\begin{aligned} \frac{d\Gamma^{D_0^*}}{dq^2} &= \frac{|V_{cb}|^2 G_F^2 M M'^2}{96\pi^3} \omega_3 \left(1 + \frac{m_l^2}{q^2}\right)^2 F(\omega)^2 \frac{(\omega + 1)^2}{3} \\ &\times \left[\left(1 - \frac{m_l^2}{2q^2}\right) \left\{ \omega_3^2 (1 - r)^2 \right\} + m_l^2 \frac{3(\omega - 1)^2 (1 + r)^2}{-2q^2} \right], \end{aligned} \quad (2.21)$$

where $\omega_3 \equiv \sqrt{\omega^2 - 1}$ and $r \equiv M'/M$ (M' being the mass of the final D^{**} meson).

By integrating the decay spectra over q^2 , we obtain the decay width. The physical region of $-q^2$ is $m_l^2 \leq -q^2 \leq (M - M')^2$. Large (Small) values of $-q^2$ correspond to the non-relativistic (relativistic) region, and its maximum (minimum) value corresponds to zero-recoil $\omega = 1$ (maximum-recoil $\omega = \omega_{\max} = (M^2 + M'^2 - m_l^2)/(2MM')$).

2.5. Values of the parameters

Our scheme contains only two parameters, m_q and K , related to the quark structure of mesons. We examine our problem for the following two choices of the numerical values:

Table I. The adopted values of quark masses m_q and of the inverse size β . Case A: $K = 0.106 \text{ GeV}^3$ is taken to be universal. Case B: K is determined from the mass spectra as $K = 0.0679$ and 0.0619 GeV^3 , respectively, for D^{**} and B mesons.

	m_n	m_c	m_b	β_{D/D^*}	β_B
Case A	0.384 GeV	1.62 GeV	4.94 GeV	0.181 GeV^2	0.194 GeV^2
Case B	0.400 GeV	1.70 GeV	5.00 GeV	0.148 GeV^2	0.151 GeV^2

Case A: The m_q are simply determined by $M_V = m_q + m_{\bar{q}}$, M_V being the masses of relevant vector mesons, B^* and D^* , in the ground S -wave states. The value of K is assumed to be universal,¹⁷⁾ that is, independent of flavor-contents of mesons, and determined from the Regge slope of the ρ meson trajectory, $\Omega^{\text{exp}} = 1.14 \text{ GeV}$, by the relation $\Omega = \sqrt{32m_n K}$.

Case B: They are determined from recent analyses of mass spectra, including effects due to the color Coulomb force.¹⁸⁾ In this case, the mixing of the ground 1S-states with excited 2S-states is shown to be a few percent for D or D^* and B in the amplitudes, and thus its effects seem to be negligible.

The actual values of the m_q and β for the respective systems are listed in Table I.

The values of $|V_{cb}|$ and the masses and the lifetimes of \bar{B}^0 and B^\pm mesons are taken from ref. 19). The masses of D^{**} are from ref. 20):

$$|V_{cb}| = 0.0395; \quad M_B = 5.279 \text{ GeV}, \quad \tau_B = 1.6 \text{ ps}, \quad M'_{D_2^*} = 2.459 \text{ GeV}, \\ M'_{D_1^{(jq=3/2)}} = M'_{D_{1*}^{(jq=1/2)}} = 2.422 \text{ GeV}, \quad M'_{D_0^*} = 2.36 \text{ GeV}. \quad (2.22)$$

§3. Results and conclusion

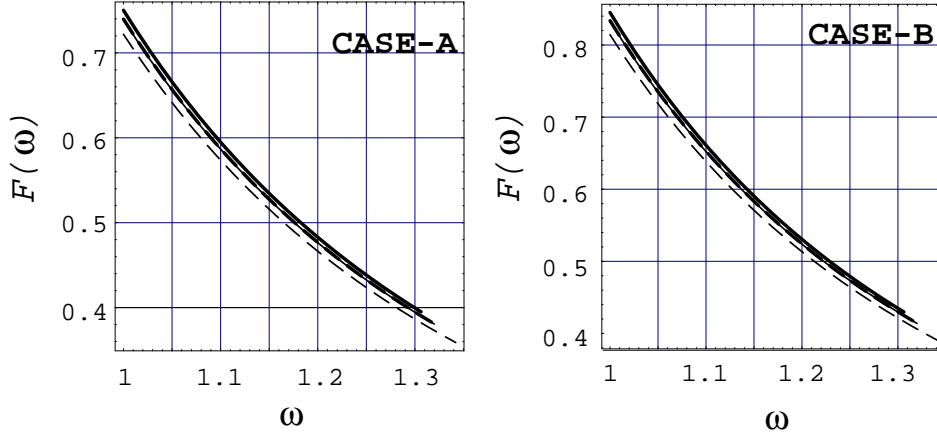


Fig. 1. F functions of $B \rightarrow D_2^*$ (thick solid line), $B \rightarrow D_1^{jq=3/2}$ (thick dashed line), $B \rightarrow D_{1*}^{jq=1/2}$ (thin solid line) and $B \rightarrow D_0^*$ (thin dashed line) transitions: The left figure is for case A, and right figure is for case B. The four lines almost coincide and are hard to be discriminated from each other in the figure. The physical regions of $-q^2$, $(M - M')^2 \geq -q^2 \geq m_l^2$, correspond to those of ω , $1 \leq \omega \leq 1.306, 1.319, 1.319, 1.342$, respectively, where we use m_l as the muon mass.

First, we display the $-q^2$ dependence of the form factor function F for the $B \rightarrow (D_2^*, D_1^{j_q=3/2}, D_{1*}^{j_q=1/2}, D_0^*)$ transitions in Fig. 1. The mass of $D_1^{j_q=3/2}$ is taken to be equal to that of $D_{1*}^{j_q=1/2}$, and thus the corresponding F functions become common. These functions are almost identical for all these transitions.

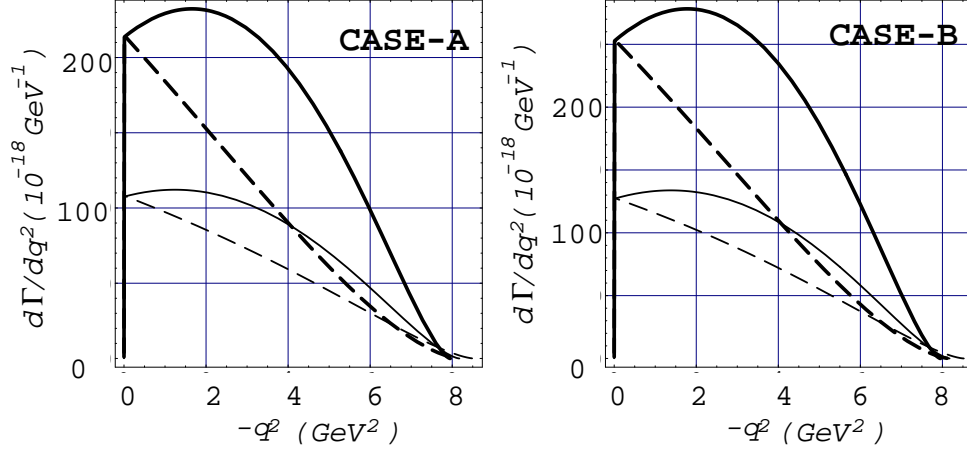


Fig. 2. Predicted spectra $d\Gamma/dq^2$ (10^{-18}GeV^{-1}) of $B \rightarrow D_2^*$ (thick solid line), $B \rightarrow D_1^{j_q=3/2}$ (thick dashed line), $B \rightarrow D_{1*}^{j_q=1/2}$ (thin solid line) and $B \rightarrow D_0^*$ (thin dashed line) transitions: The left figure is for case A, and the right figure is for case B.

By using these F , we can predict the decay spectra, given in Fig. 2. As can be seen in this figure, the differential widths in relativistic region are much larger than those in non-relativistic region, since the decay amplitude vanishes at the zero-recoil point due to the on-shell condition of the final D^{**} mesons,^{*)} and the widths in the non-relativistic region near zero-recoil point are suppressed comparatively to those in relativistic region. As discussed in the appendix, in HQET, effectively the same spinor WF as those in COQM are used for the relevant processes with no clear reason, and accordingly the models basing on HQET also predicts the comparatively larger differential widths in relativistic region. This suggests that the quantitative estimation of the quark confined effects is crucially important to estimate the decay widths.

By integrating out this spectra, the theoretical branching ratios Br_{th} are obtained and given in Table II. There they are compared with the experimental Br_{exp} , together with the predictions of the other theoretical models (ISGW model,²¹⁾ CNP model,²²⁾ and SISM model²³⁾). Experimental values of Br ., Br_{exp} , for $\bar{B} \rightarrow D_{1*}^{j_q=1/2} l \bar{\nu}_l$ and $\bar{B} \rightarrow D_0^* l \bar{\nu}_l$ have not yet been obtained. The present Br_{exp} have large errors and seem to be mutually inconsistent. Our predicted values of Br_{th} are somewhat different from the other models. Future experiments will select the correct model.

^{*)} The P -wave WF of D^{**} includes the factor x_μ , and thus the overlapping of the initial and final space-time WF becomes proportional to v_μ or v'_μ . At zero-recoil point v_μ becomes equal to v'_μ , and their contractions to the polarization vectors of the D^{**} mesons vanish because of Lorentz condition.

Table II. Predicted branching ratios Br_{th} compared with experimental values. Br_{th} without (with) brackets corresponds to case A (B). The lifetime of the B meson is taken as $\tau_B = 1.6$ ps. The experimental values of Br., Br_{exp} , have not yet been obtained for the decays $\bar{B} \rightarrow D_{1*}^{jq=1/2} l \bar{\nu}_l$ and $\bar{B} \rightarrow D_0^* l \bar{\nu}_l$. The ISGW model uses the LS coupling scheme, and the values given in the columns for $\bar{B} \rightarrow D_1^{jq=3/2} l \bar{\nu}_l$ and $\bar{B} \rightarrow D_{1*}^{jq=1/2} l \bar{\nu}_l$ are those for 1P_1 and 3P_1 states, respectively.

Br.	$\bar{B} \rightarrow D_2^* l \bar{\nu}_l$	$\bar{B} \rightarrow D_1^{jq=3/2} l \bar{\nu}_l$	$\bar{B} \rightarrow D_{1*}^{jq=1/2} l \bar{\nu}_l$	$\bar{B} \rightarrow D_0^* l \bar{\nu}_l$
Exp.				
OPAL	$2.3 \pm 0.9\%$ (\bar{B}^0) $0.9 \pm 0.4\%$ (B^-)	$2.1 \pm 0.9\%$ (\bar{B}^0) $2.1 \pm 0.7\%$ (B^-)		
CLEO	$< 1.0\%$ (B^-)	$0.49 \pm 0.14\%$ (B^-)		
ALEPH	$< 0.15 \sim 0.20\%$	$0.74 \pm 0.16\%$		
Theor.				
COQM	$0.31(0.38)\%$	$0.19(0.22)\%$	$0.15(0.18)\%$	$0.11(0.14)\%$
ISGW	0.13%	$(0.57\% \text{ for } ^1P_1)$	$(0.30\% \text{ for } ^3P_1)$	0.14%
CNP	0.24%	0.12%	0.09%	0.07%
SISM	0.12%	0.087%	0.036%	0.027%

Finally, we would like to remark on the treatment of B decay with HQET in comparison to COQM. For describing the decay to the ground and the first excited D mesons, the three independent Isgur-Wise functions, $\xi(\omega)$, $\tau_{3/2}(\omega)$ and $\tau_{1/2}(\omega)$, which are not derivable only in the general framework of HQET, are necessary. In COQM, these Isgur-Wise functions are predicted with no free parameters. Heavy-to-light transitions, such as $B \rightarrow \rho$, also cannot be described with HQET, while in the case of COQM they can be treated on the same footing as heavy-to-heavy transitions. The treatment using HQET is considered to become less reliable, in principle, for more excited states, since the approximation of the HQS limit, $m_{lq} \ll m_Q$ (m_{lq} is the mass of the light degrees of freedom), becomes worse for increasingly excited states. In COQM the excited states can be treated on the same footing as for the ground states.

In this paper we have analyzed the semi-leptonic B meson decays to the first excited D mesons in the framework of COQM. All the parameters in COQM have been determined by the analyses of mass spectra, and thus the results in this paper are pure predictions, with no free parameters. The framework of COQM has been found to be very effective in describing general weak decays of ground-to-ground transitions. Whether it is also effective for describing the decay of ground-to-excited transitions is a very interesting question. However, the present experimental data cannot answer this question.

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Appendix A

—— Derivation of the Form Factor Relation and
Comparison of Our Spinor Overlapping Calculation with HQET ——

Here we compare our overlapping calculation of BW spinors with that using HQET by Falk et al.¹⁶⁾ In Ref. 16), the WF for the final mesons with $j_q = 1/2$ are taken as

$$\begin{aligned} (1 - iv' \cdot \gamma) & \quad \text{for} \quad D_0^* \\ \gamma_5(i\tilde{\epsilon} \cdot \gamma)(1 - iv' \cdot \gamma) & \quad \text{for} \quad D_{1*}^{j_q=1/2}, \end{aligned} \quad (\text{A}\cdot 1)$$

where we omit the overall normalization factor $1/2\sqrt{2}$. The WF of the final mesons with $j_q = 3/2$ are taken as

$$\begin{aligned} \sqrt{\frac{3}{2}}\gamma_5 \left[\tilde{\epsilon}_\lambda + \frac{1}{3}(i\gamma_\lambda + v'_\lambda)i\tilde{\epsilon} \cdot \gamma \right] (1 - iv' \cdot \gamma) & \quad \text{for} \quad D_1^{j_q=3/2} \\ -i\gamma_\nu \tilde{\epsilon}_{\lambda\nu}(1 - iv' \cdot \gamma) & \quad \text{for} \quad D_2^*. \end{aligned} \quad (\text{A}\cdot 2)$$

In the overlapping calculation of the initial B and final D^{**} mesons, the suffix λ in Eq. (A·2) is contracted with the velocity of the initial B meson, v_λ , for no clear reason, and Eq. (A·2) is rewritten as

$$\begin{aligned} \sqrt{\frac{3}{2}}\gamma_5 \left[\tilde{\epsilon} \cdot v + \frac{1}{3}(iv \cdot \gamma - \omega)i\tilde{\epsilon} \cdot \gamma \right] (1 - iv' \cdot \gamma) & \quad \text{for} \quad D_1^{j_q=3/2} \\ -i\gamma_\nu \tilde{\epsilon}_{\lambda\nu}(1 - iv' \cdot \gamma)v_\lambda & \quad \text{for} \quad D_2^*. \end{aligned} \quad (\text{A}\cdot 3)$$

In COQM the overlapping of space-time WF, O_λ^x , is proportional to $(v_\lambda - \omega v'_\lambda)$, as shown in Eq. (2·8). Contraction of the second term $\omega v'_\lambda$ with the spinor overlapping $O_{\mu\lambda}^s$ vanishes, due to the on-shell condition of the final D^{**} mesons, and the contraction of the v_λ term only contributes to the effective current J_μ . Thus, the above contraction of v_λ in HQET is naturally explained in our scheme.

By including this contraction with v_λ , in the overlapping calculation Eq. (2·8) of COQM, the WF for the final mesons with $j_q = 1/2$ are taken as

$$\begin{aligned} (1 + iv' \cdot \gamma)(-i\gamma_\nu) \frac{\delta_{\nu\lambda} + v'_\nu v'_\lambda}{-\sqrt{3}}(v_\lambda - \omega v'_\lambda) &= \frac{iv \cdot \gamma + \omega}{\sqrt{3}}(1 - iv' \cdot \gamma) \\ &= \frac{1 + \omega}{\sqrt{3}}(1 - iv' \cdot \gamma) \quad \text{for} \quad D_0^*, \end{aligned} \quad (\text{A}\cdot 4)$$

where in the last equality the term $iv \cdot \gamma$ in the left hand side is replaced by 1, because of the on-shell factor $(1 + iv \cdot \gamma)$ of the initial B meson spinor WF. Equation (A·4) coincides with the first equation of (A·1) except for the factor $(1 + \omega)/\sqrt{3}$.

Similarly, we have

$$(1 + iv' \cdot \gamma) \left[(-i\gamma_\nu \frac{1}{\sqrt{2}} \epsilon_{\nu\lambda\alpha\beta} v'_\alpha \tilde{\epsilon}_\beta) \sqrt{\frac{2}{3}} + (-\gamma_5 \tilde{\epsilon}_\lambda) \left(-\sqrt{\frac{1}{3}}\right) \right] (v_\lambda - \omega v'_\lambda)$$

$$\begin{aligned}
&= -\frac{1}{\sqrt{3}}[\epsilon_{\nu\lambda\alpha\beta}i\gamma_\nu v_\lambda v'_\alpha \tilde{\epsilon}_\beta - \gamma_5 \tilde{\epsilon} \cdot v](1 - iv' \cdot \gamma) \\
&= \frac{1+\omega}{\sqrt{3}}\gamma_5 i\tilde{\epsilon} \cdot \gamma(1 - iv' \cdot \gamma) \quad \text{for } D_1^{j_q=1/2}
\end{aligned} \tag{A.5}$$

where in the last equality we have used the formula $\epsilon_{\nu\lambda\alpha\beta}i\gamma_\nu = -i\gamma_5\gamma_\lambda\gamma_\alpha\gamma_\beta + i\gamma_5\gamma_\lambda\delta_{\alpha\beta} + i\gamma_5\gamma_\beta\delta_{\lambda\alpha} - i\gamma_5\gamma_\alpha\delta_{\lambda\beta}$. Equation (A.5) coincides with the second equation of (A.1), except for the factor $(1+\omega)/\sqrt{3}$. The conventional Isgur-Wise function $\tau_{1/2}$ is defined in terms of WF Eq. (A.1). Thus, $\tau_{1/2}$ is represented by our F function as

$$\tau_{1/2} = F(\omega)(\omega + 1)/\sqrt{3}. \tag{A.6}$$

Similarly, by including the contraction with v_λ mentioned above, the WF for the final mesons with $j_q = 3/2$ are given by

$$\begin{aligned}
&(1 + iv' \cdot \gamma) \left[(-i\gamma_\nu \frac{1}{\sqrt{2}}\epsilon_{\nu\lambda\alpha\beta}v'_\alpha \tilde{\epsilon}_\beta)(-\sqrt{\frac{1}{3}}) + (-\gamma_5 \tilde{\epsilon}_\lambda)(-\sqrt{\frac{2}{3}}) \right] (v_\lambda - \omega v'_\lambda) \\
&= \sqrt{\frac{3}{2}}\gamma_5 \left[\tilde{\epsilon} \cdot v + \frac{iv \cdot \gamma - \omega}{3}i\tilde{\epsilon} \cdot \gamma \right] (1 - iv' \cdot \gamma) \quad \text{for } D_1^{j_q=3/2},
\end{aligned} \tag{A.7}$$

$$(1 + iv' \cdot \gamma)(-i\gamma_\nu)\tilde{\epsilon}_{\nu\lambda}(v_\lambda - \omega v'_\lambda) = -i\gamma_\nu\tilde{\epsilon}_{\nu\lambda}(1 - iv' \cdot \gamma)v_\lambda \quad \text{for } D_2^* \tag{A.8}$$

These are exactly the same as Eq. (A.3) in HQET, and the Isgur-Wise function $\tau_{3/2}$ is given as

$$\tau_{3/2} = F(\omega). \tag{A.9}$$

Equations (A.6) and (A.9) are Eq. (2.13) in the text.

The essential point for obtaining the same form factor relation in COQM as in HQET is use of the BW spinor as spin WF, which is equivalent to direct product of the “free” Dirac spinors of constituent quarks and antiquarks. On the other hand, in HQET only the heavy quark is argued to be on shell, while the momentum of the light quark (or light degrees of freedom) is not defined clearly. However, as discussed previously,²⁾ fixing the velocity of the heavy quark, $v_{Q\mu}$, to be equal to the meson velocity, v_μ , necessarily leads to the velocity of the light quark, $v_{q\mu}$, also being equal to v_μ , which implies that the light quark also is on shell. Accordingly, the BW spinor WF is also implicitly used in HQET.

Finally, we comment on the SISM model.²³⁾ In this model, to obtain the same form factor relation, the spinor WF given by Falk et al. is applied. Their Isgur-Wise functions $\tau_{1/2}$ and $\tau_{3/2}$, calculated using the overlapping of the initial and final WF in NRQM, are argued to vanish at the zero-recoil point, $\omega = 1$, since the initial and final WF are orthogonal in the HQS limit in this case. However, this argument is not correct. In COQM, the overlapping of space-time WF, O_λ^x , is given by $O_\lambda^x \propto F(\omega)(v_\lambda - \omega v'_\lambda)$. At the zero-recoil point, where $v_\lambda = v'_\lambda$ and $\omega = 1$, O_λ^x vanishes due to the factor $(v_\lambda - \omega v'_\lambda)$. However, the “Isgur-Wise function” $F(\omega)$ (or $F(\omega)(1 + \omega)/\sqrt{3}$) does not vanish. The factor $(v_\lambda - \omega v'_\lambda)$ already appeared for the spinor overlapping calculation in HQET, as was discussed above, and this seems

to imply in the calculation of the SISM model that the factor $(v_\lambda - \omega v'_\lambda)$ is used twice. This may be the reason why their branching ratios are much smaller than those obtained with the ISGW model, although both models are based on NRQM.

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